arXiv:hep-lat/0409082 v1 15 Sep 2004

Analysis of $N^\star$ spectra using matrices of correlation functions based on irreducible baryon operators

LHP Collaboration: S. Basak[a*], R. Edwards[b], R. Fiebig[c], G.T. Fleming[bd], U.M. Heller[e], C. Morningstar[f], D. Richards[b], I. Sato[a] and S. Wallace[a]

[a]Department of Physics, University of Maryland, College Park, MD 20742, USA

[b]Thomas Jefferson National Accelerator Facility, Newport News, VA 23606, USA

[c]Department of Physics, Florida International University, Miami, FL 33199, USA

[d]Sloane Physics Lab, Yale University, 217 Prospect St, New Haven, CT 06520, USA

[e]American Physical Society, One Research Road, Ridge, NY 11961, USA

[f]Department of Physics, Carnegie Mellon University, Pittsburgh, PA 15213, USA

We present results for ground and excited-state nucleon masses in quenched lattice QCD using anisotropic lattices. Group theoretical constructions of local and nonlocal straight-link irreducible operators are used to obtain suitable sources and sinks. Matrices of correlation functions are diagonalized to determine the eigenvectors. Both chi-square fitting and Bayesian inference with an entropic prior are used to extract masses from the correlation functions in a given channel. We observe clear separation of the excited state masses from the ground state mass. States of spin $\geq 5/2$ have been isolated by use of G_2 operators.

1. INTRODUCTION

Reproducing the spectrum of baryon resonances with spin-1/2 and spin-3/2 and both parities is an important test of lattice QCD. For that we require three-quark operators that transform irreducibly under the spinorial rotation group of the lattice [1]. Local and nonlocal straight-link operators corresponding to irreducible representations (IRs) $G_{1g,u}$, $H_{g,u}$ and $G_{2g,u}$ are constructed to obtain suitable sources and sinks [2]. Here we analyze $N^\star$ spectra using these operators.

To determine the $N^\star$ excited states, a matrix of correlation functions is computed in the quenched approximation to QCD using irreducible baryon interpolating operators $\overline{B}_i(\vec{x},t)$ of definite quantum numbers,

$$C_{ij}(t) = \sum_{\vec{x}} \langle 0|T\big(B_i(\vec{x},t)\overline{B}_j(\vec{0},0)\big)|0\rangle. \qquad (1)$$

The $C_{ij}(t)$ matrices for G_1 and H states for each parity are constructed using local operators, smeared local operators and smeared straight-link operators. For G_2, we have one operator which is of smeared straight-link type (Tables 1, 3 in [2]).

The computation of masses of the lowest-lying resonances is based on the variational method applied to the matrix of correlation functions. In this paper we solve the generalized eigenvalue equation,

$$C_{ij}(t)\, V_j^{(\alpha)}(t) = \lambda^{(\alpha)}(t,t_0)\, C_{ij}(t_0)\, V_j^{(\alpha)}(t) \qquad (2)$$

and determine eigenvectors $V^{(\alpha)}(t)$ for each t, with t_0 close to the source time. Then the masses of $N^\star$ states correspond to the eigenvalues of Eq. (2): $\lambda^{(\alpha)}(t,t_0) \xrightarrow[t\gg t_0]{} e^{-m^{(\alpha)}(t-t_0)}$ [3]. The effective masses are determined from

$$m_{\rm eff}^{(\alpha)} = \ln\left[\frac{\lambda^{(\alpha)}(t,t_0)}{\lambda^{(\alpha)}(t+1,t_0)}\right] \xrightarrow[t\gg t_0]{} m^{(\alpha)}. \qquad (3)$$

Another way to extract spectrum information is to calculate the spectral mass density $\rho(\omega)$ from

*Presented by S. Basak

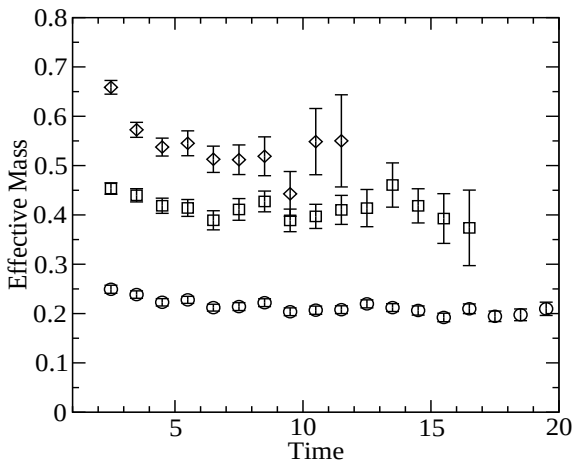

Figure 1. G_{1g} effective masses for a selected few low-lying states.

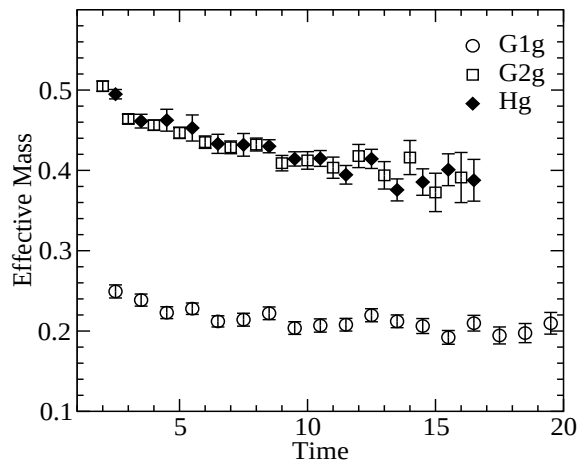

Figure 2. Lowest positive parity effective masses for the IRs G_{1g}, H_g and G_{2g}.

lattice correlation functions using the Maximum Entropy Method (MEM) [4],

$$C(t,t_0) \longrightarrow \int d\omega\, \rho(\omega) e^{-\omega(t-t_0)}. \tag{4}$$

One of the advantages of MEM is that it utilizes data on a wide range of available time slices and has been shown to yield results even for noisy data [4]. This feature may be helpful in extracting masses of excited states.

2. RESULTS

We use an ensemble of 287 quenched, anisotropic $16^3 \times 64$ lattices with renormalized anisotropy $\xi = 3.0$ and $\beta = 6.1$, corresponding to $a_t^{-1} = 6.0$ GeV [5]. We use the anisotropic Wilson action. The parameters of the Wilson fermion action are tuned nonperturbatively so as to satisfy the continuum dispersion relation $E(\mathbf{p})^2 = E(0)^2 + c(\mathbf{p})^2\mathbf{p}^2$ at a pion mass $m_\pi \simeq 500$ MeV. To improve the coupling of operators to the lower mass states we employ gauge-covariant smearing of the quark fields on both source and sink: $\tilde{\psi}(x) = (1 + \sigma^2\tilde{\Delta}(\tilde{U})/4N)^N\psi(x)$, where $\tilde{\Delta}(\tilde{U})$ is the three dimensional Laplacian and $\tilde{U}$ denotes APE-smeared $SU(3)$ link variables. The parameters used to smear the quark fields are $\sigma = 3.6$ and $N = 32$.

The effective masses are calculated from 10×10 G_{1g} matrices with $t_0 = 2$. In Fig. 1 we choose to show a few low-lying states that are clearly separated. However, the details of the states above the ground state are under study. The plot shows a good plateau for the ground state and statistically significant splittings for a couple of excited states.

In Figs. 2 and 3 we have collected the effective mass plots of the lowest states of both parities for G_1, G_2 and H. The ratio of lowest masses for G_{1g} and G_{1u} is roughly in accordance with experiment, for spin-1/2 states, the G_{1u} mass being higher. The effective masses for $H_{g,u}$ are obtained using a 7×7 matrices of correlation functions. The lowest negative parity H_u state has smaller mass than the lowest positive parity H_g state. This is compatible with the pattern found in nature for spin-3/2. However, the masses of H_u and G_{1u} overlap within errors. Our result for G_2 masses also reveals reasonable separations of the G_{2g} and G_{2u} masses, G_{2g} being lower. From Fig. 2, it is evident that the effective mass for H_g (allowed spin $3/2^+$, $5/2^+$, $\cdots$) is very similar to that for G_{2g} (allowed spin $5/2^+$, $7/2^+$, $\cdots$). These states are orthogonal. One possibility for this is that the lowest H_g state has spin-$3/2^+$ and its mass is accidentally close to that of the lowest G_{2g} state. Another possibility is that the lowest H_g state is spin-$5/2^+$, in which case the same

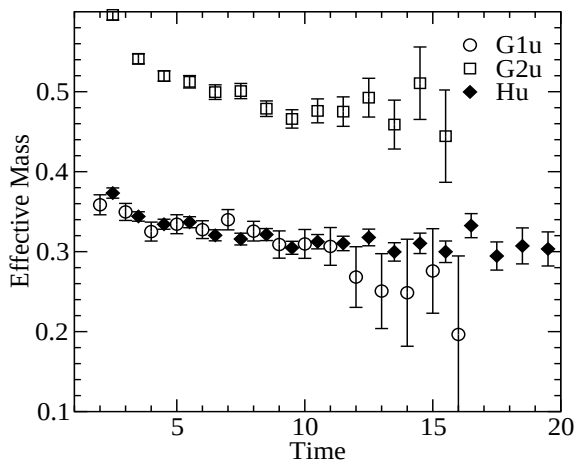

Figure 3. Lowest negative parity effective masses for the IRs G_{1u}, H_u and G_{2u}.

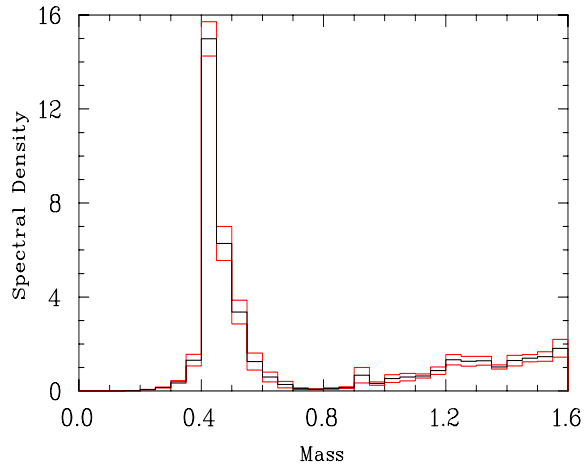

Figure 4. Example of a spectral density function from a MEM analysis of G_{2g} correlation functions.

state must be present in H_g and G_{2g}, but not in G_{1g}. Study over different values of lattice spacing is required to decide.

Finally, we present the G_{2g} MEM spectral function in Figure 4. We find that the peak of the spectral density roughly corresponds to the effective mass value.

In Table 1 we summarize our preliminary estimates of the lowest masses for the different representations extracted from single-exponential fits to the $\lambda^{(\alpha)}(t, t_0)$ of Eqn. 1. The effective masses for the lowest states of G_1, H and G_2 for both parities, whether obtained from the variational method or preliminary MEM analysis, show a spectrum of distinct $N^\star$ masses. However, the behavior of the spectrum with m_π and the sensitivity of the spectrum to variations in the lattice volume has yet to be studied.

Table 1
Numerical estimates from single-exponential fit to $\lambda^{(\alpha)}(t, t_0)$ corresponding to the lowest masses of $N^\star$ spectra.

IRs	Fit range	m_{eff}	$\sim$ Mass (MeV)
G_{1g}	9 – 20	0.208 (4)	1250
G_{1u}	8 – 12	0.321 (4)	1930
H_g	9 – 12	0.410 (2)	2460
H_u	6 – 15	0.315 (4)	1890
G_{2g}	9 – 14	0.409 (7)	2450
G_{2u}	8 – 15	0.475 (7)	2850

This work is supported by US National Science Foundation under Awards PHY-0099450 and PHY-0300065, and by US Department of Energy under contract DE-AC05-84ER40150 and DE-FG02-93ER-40762.

REFERENCES

1. C. Morningstar *et al.*, Nucl. Phys. B (Proc. Suppl.) 129 (2004) 236 and these proceedings
2. I. Sato *et al.* Nucl. Phys. B (Proc. Suppl.) 129 (2004) 209 and these proceedings
3. C. Michael, Nucl. Phys. B259 (1985) 58; M. Lüscher *et al.*, Nucl. Phys. B339 (1990) 222; C.R. Alton *et al.*, Phys. Rev. D47 (1993) 5128
4. H.R. Fiebig, Phys. Rev. D65 (2002) 094512
5. R.G. Edwards *et al.*, Nucl. Phys. B (Proc. Suppl.) 119 (2003) 305